# SPECIALIZED KNOWLEDGE OF THE FUTURE TEACHER OF EARLY CHILDHOOD EDUCATION TO REFLECT ON ALGEBRAIC THINKING IN A PROFESSIONAL TRAINING TASK OF CLASSIFICATION[1]

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**ABSTRACT:** This article aims to identify elements of the specialized knowledge mobilized by future teachers of Early Childhood Education in initial training on algebraic thinking when they develop a professional formative classification task. The selection was made because we believe that classification is a learning topic in Early Childhood Education that can foster logical-mathematical thinking, which in turn allows us to reflect on and develop algebraic thinking by considering the relationships that are evident in the classification process. Therefore, the theoretical framework we used for this analysis is Mathematics Teacher's Specialized Knowledge (MTSK). The methodology used was a Teaching Experiment, applied in two initial training classes for early childhood education teachers at the University of Seville. Our results suggest the possibility of advancing discussions about the mathematical knowledge early childhood education teachers need regarding the specificities of classification.

**KEYWORDS:** Mathematics education. Teacher education. Initial training. Training tasks.

## CONHECIMENTO ESPECIALIZADO DO FUTURO PROFESSOR DE EDUCAÇÃO INFANTIL PARA REFLETIR SOBRE O PENSAMENTO ALGÉBRICO EM UMA TAREFA FORMATIVA PROFISSIONAL DE CLASSIFICAÇÃO

**RESUMO:** Este artigo tem como objetivo identificar elementos dos conhecimentos especializados mobilizados por futuros professores de Educação Infantil em uma formação inicial sobre o pensamento algébrico quando desenvolvem uma tarefa formativa profissional de classificação. A seleção foi feita porque consideramos que a classificação é um conteúdo de aprendizagem da Educação Infantil que pode impulsionar o pensamento lógico matemático, que por sua vez nos permite refletir e desenvolver o pensamento algébrico, por considerar as relações que estão em evidência no processo de classificação. Assim, a referência teórica que usamos para analisar é o Mathematics Teacher's Specialized Knowledge (MTSK). A metodologia utilizada foi o Teaching Experiment, aplicado em duas classes de

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formação inicial para professores de educação infantil da Universidade de Sevilha. Apresentamos como resultados a possibilidade de avanço nas discussões sobre quais conhecimentos matemáticos os professores de educação infantil precisam ter sobre as especificidades da classificação.

**PALAVRAS–CHAVE:** Educação matemática. Formação de professores. Formação inicial. Tarefas formativas.

## Introduction

This article is the result of research developed at the University of Seville in the years 2023 and 2024. The aim was to identify elements of the specialized knowledge mobilized by prospective Early Childhood Education teachers in the context of a formative task focused on algebraic thinking as they engage in a professional formative task of classification. We know that initial teacher education is related to the educator who conducts it, the proposed training curriculum, and the characteristics and specificities of the students undergoing teacher education for early childhood (Joglar-Prieto, Liñán-García & Contreras[5]; Barreira-Castarnado, Contreras; Liñán-García; Muñoz-Catalán[6]). When we consider the target group of initial teacher education and algebraic thinking, we find that there is limited research that enables reflection on the essential knowledge related to this theme, as noted by Beck and Silva[7]; and Pincheira and Alsina[8].

Therefore, we consider that the reflections on algebraic thinking, despite having started with the research of LaCampagne et al.[9] which brings reflections on investigations with the theme of algebraic thinking, demonstrating how it can be

[5] Joglar Prieto, N., Liñán García, M. d. M., & Contreras González, L. C. (2022). MTSK in the initial training of primary teachers. In Research on specialist knowledge of mathematics teacher. (MTSK): 10 años de camino (pp. 207–222). Dykinson.
[6] Barrera Castarnado, V. J., Contreras González, L. C., Muñoz Catalán, M. C., & Liñán García, M. M. (2024). Specialized teacher knowledge: A teaching experiment focused on a formative task in geometry. AIEM - Advances in Research in Mathematics Education, 26, 1–19. https://doi.org/10.35763/aiem26.5359
[7] Beck, V. C., & Silva, J. A. (2015). The State of the Art in Research on Algebraic Thinking with Children. *REVEMAT,* 10, 197–208.
[8] Pincheira, Nataly, & Alsina, Ángel. (2021). Teachers' mathematics knowledge for teaching early algebra: A systematic review from the MKT perspective. *Mathematics,* 9(20), 2590. Available at: https://dugi-doc.udg.edu/bitstream/handle/10256/20017/033981.pdf?sequence=1. Accessed in Dec. 2022.
[9] Lacampagne C. B., Blair W., Kaput J. J. (1995) (Eds.). *The algebra initiative colloquium: Vol 1: Plenary and reactor papers*. Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement. Vol 2: Working group papers ED385437.pdf Retrieved from https://files.eric.ed.gov/fulltext/ED385436.pdf.

taught, and on the other hand, the NCTM[10] document shows a curricular reflection, listing which skills and contents should be developed regarding algebraic thinking. We note how these reflections on the knowledge of algebraic thinking have advanced considerably across different educational stages. However, there is still little research connecting initial teacher education with the theoretical model of Mathematics Teacher's Specialized Knowledge (MTSK), algebraic thinking, and its relationship with classification. Most of the research we found focuses on other content areas, such as geometry, numbers, and operations (Barreira-Castarnado; Contreras; Liñán García; Muñoz-Catalán[11]), making it evident that the development of research on algebraic thinking specifically in the context of classification—as a precursor to this content within initial teacher education using the MTSK theoretical model—is an unexplored topic. This is confirmed in the review study by Moriel Junior[12], which, in analyzing investigations based on the MTSK model, does not include data on research addressing this specific thematic approach. Moreover, we consider that, although there are other formative studies on the proposed theme, the use of the MTSK model encourages more focused reflections on teacher education with specificity in a mathematical topic—in the case of this article, the development of algebraic thinking.

We consider classification to be a precursor of algebraic thinking, as it results from the establishment of a relationship within a set. In this way, such a relationship generates a subset of the initial set. When the relationship is one of equivalence, these subsets are disjoint, and their union constitutes the original set. Based on this, we consider classification important for the formation of sets, which are the foundation for the study of algebra, since it is through sets that one can explore properties, characteristics, and the operations involving them. Thus, classifying in early childhood education allows the development of perceptions regarding the structural characteristics of mathematical elements, which are present in set formation and, consequently, in algebraic thinking. For this reason, classification enables the identification of similarities and differences between the objects being

[10] NCTM [National Council of Teachers of Mathematics]. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Algebra/

[11] See footnote 5

[12] Moriel Junior, J. G. (2021). Specialized mathematics teacher knowledge (MTSK) in the Web of Science up to 2020. *Zetetike*, 29(00), e021022. https://doi.org/10.20396/zet.v29i00.8660030

classified (NCTM[13], Kaput[14], Blanton & Kaput[15], Carraher & Schliemann[16]). Therefore, the choice of focusing on classification content was made because we consider that the relationships established in classification—specifically in the formation of sets—allow for the identification of properties essential to the development of algebraic thinking.

Another justification is that algebraic thinking has been presented in curriculum documents from different countries, showing the development of the classification content. Such evidence is found in documents from the NCTM[17] and from Brazil[18], as well as in the research by Acosta and Alsina[19], who mention that countries like Singapore, Australia, and New Zealand have incorporated the teaching of algebra in early childhood education.

Considering that the data collected by these studies indicate and reaffirm the need to develop research on teacher education for teaching algebraic thinking, we focused our attention on this investigation.

Thus, this research aims to identify elements of specialized knowledge mobilized by prospective Early Childhood Education teachers in initial training regarding algebraic thinking when they engage in a professional formative task of classification.

Accordingly, the research question we aim to answer in this article is: What specialized knowledge focused on algebraic thinking is revealed by prospective early childhood teachers through a professional formative classification task? Based on this, we have organized this article by presenting the theoretical framework used,

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[13] See footnote 9
[14] Kaput, J. J. (2008) *What is algebra? What is algebraic reasoning***?** In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), Algebra in the Early Grades (pp. 5–17). New York, NY: NCTM & Lawrence Erlbaum Associates.
[15] Blanton, M., & Kaput, J. (2011) *Functional Thinking as a Route into Algebra in the Elementary Grades.* In J. Cai & E. Knuth (Eds.), *Early Algebraization, Advances in Mathematics Education.* Berlin Heidelberg: Springer-Verlag. https://doi.org/10.1007/978-3-642-17735-4_2.
[16] Carraher, D. W., & Schliemann, A. D.(2019) *Early algebraic thinking and the US mathematics standards for grades K to 5. Infancia y Aprendizaje: Journal for the Study of Education and Development*, 42(3), 479–522. https://doi.org/10.1080/02103702.2019.1638570.
[17] See footnote 9.
[18] Brazil. Ministry of Education. (2018) *Base Nacional Comum Curricular* [National Common Curricular Base]. Brasília: MEC.
[19] Acosta, Y., & Alsina, Á. (2020) *Learning patterns at three years old: Contributions of a learning trajectory and teaching itinerary. Australasian Journal of Early Childhood*, 45(1), 14–29. https://doi.org/10.1177/1836939119885310,

the methodology describing how the investigation was conducted, the data analysis, and the final considerations.

**Context the theoretical framework used**

We present this theoretical framework in two parts: the first explores how classification is related to the development of algebraic thinking, and the second introduces the Mathematics Teacher's Specialized Knowledge (MTSK) by Carrillo et al.[20], which will serve as the foundation for our analyses.

**But why focus on algebraic thinking through classification?**

The theme of algebraic thinking began with the studies of LaCampagne et al.[21], and we found it relevant to include in this article because it was through that scientific communication that initial reflections on algebraic thinking and teaching began. In the NCTM[22] curriculum document, the first reflections on the curricular topics of algebraic thinking for education are proposed.

The NCTM[23] states that, in their learning process, students should:

> Understand patterns, relationships, and functions
> Represent and analyze mathematical situations and structures using algebraic symbols
> Use mathematical models to represent and understand quantitative relationships
> Analyze change in various contexts

And by introducing these learning standards, the NCTM[24] encourages reflection on the characteristics of algebraic thinking for the development of student learning. Authors such as Squalli[25] expand on NCTM's[26] ideas and present elements for the development of algebraic thinking. It is worth noting that although the investigation does not bring specific reflections on algebraic thinking in early

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[20] Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). *The Mathematics Teacher's Specialised Knowledge (MTSK) Model*. *Research in Mathematics Education*. https://doi.org/10.1080/14794802.2018.1479981
[21] See footnote 8
[22] See footnote 9
[23] See footnote 9
[24] See footnote 9
[25] Squalli, H. (2003). *Tout, tout, tout, vous saurez tout sur l'algèbre*. Trois-Rivières: Éditions Bande Didactique.
[26] See footnote 9

childhood education, it does provide important elements for building a theoretical construct. According to Squalli[27], it is important to develop certain skills to deepen algebraic thinking. These include: i) analytical thinking; ii) construction, interpretation, and validation of algebraic models, which may be involved in real-world or abstract—but essentially mathematical—situations; iii) manipulation of algebraic situations with defined rules; iv) abstraction and generalization of relationships, rules, and algebraic structures.

In light of the above, Blanton and Kaput[28] define algebraic thinking as: "the process by which students generalize mathematical ideas from a set of particular cases, establish those generalizations through argumentative discourse, and express them in increasingly formal ways appropriate to their age".

Therefore, in this research, we chose classification as our mathematical theme and highlight that it allows for the establishment of connections, enabling us to understand what a set is and the implications for the formation of a set. In early childhood education, the foundational knowledge for set formation is classification. According to Lorenzato[29], for a teacher to develop basic concepts in early childhood education, it is necessary to explore seven mental fields, one of which is classification. The author describes classification as "the act of separating into categories according to similarities or differences." This perception involves selecting and grouping objects into classes based on a rule or principle.

Complementing the idea that classification is related to set relationships, the NCTM[30] brings specific guidance for Early Childhood Education:

> to sort, classify, and order objects by size, number, and other properties;
> to recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numerical patterns, and to translate from one representation to another;
> to analyze how repeating and growing patterns are generated.

In this way, we consider that classification allows the development of knowledge about what a set is, what its properties are, and what its relationships

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[27] See footnote 23
[28] Blanton, M., & Kaput, J. (2005). *Characterizing a Classroom Practice That Promotes Algebraic Thinking*. *Journal for Research in Mathematics Education*, 36(5), 413.
[29] Lorenzato, S. (2008). *Educação Infantil e Percepção Matemática*. Campinas/SP: Autores Associados, pp. 23–29.
[30] See footnote 9

are. The development of flexible thinking is what enables us to develop skills related to algebraic thinking.

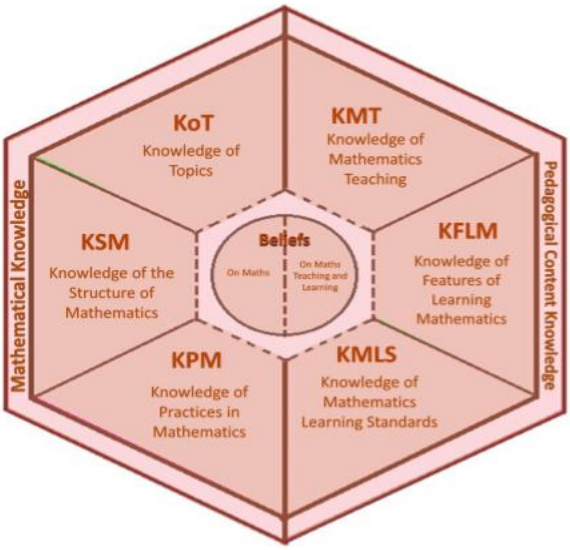
Other studies, such as those by Carraher and Schliemann[31], lead us to the hypothesis that it is through these generalizations that the processes of set formation occur, based on classification criteria. For this reason, we consider classification a precursor mental skill for the development of algebraic thinking.

## Mathematics Teacher's Specialised Knowledge (MTSK)

For the purpose of this study, we used the theoretical model Mathematics Teacher's Specialized Knowledge (MTSK), by Carrillo et al.[32], as we consider it appropriate given the specificities established by its domains and subdomains.

As shown in Figure 1, the MTSK model is organized into two main domains: MK – Mathematical Knowledge and PCK – Pedagogical Content Knowledge.

**Figure 1** – MTSK Model




Source: Carrillo[33]

[31] See footnote 15
[32] Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). *The Mathematics Teacher's Specialized Knowledge (MTSK) Model*. *Research in Mathematics Education*. https://doi.org/10.1080/14794802.2018.1479981
[33] See footnote 30

We can observe that each domain consists of three subdomains: KoT – Knowledge of Topics, KSM – Knowledge of the Structure of Mathematics, KPM – Knowledge of Mathematical Practice, KMT – Knowledge of Mathematics Teaching, KFLM -Knowledge of Features of Learning Mathematics and KMLS – Knowledge of Mathematics Learning Standards. At the center are teachers' beliefs. It is important to emphasize that, despite this organization, there is an interrelationship among these components, which together form the specialized knowledge of the mathematics teacher.

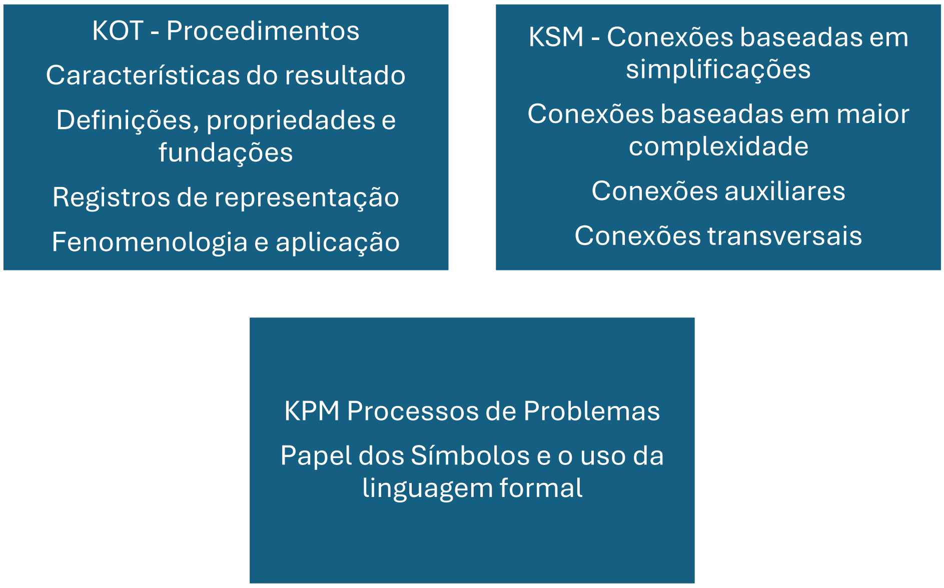
Thus, we briefly present below what each of these components refers to:

- KoT – Knowledge of Topics refers to the properties of mathematical knowledge, representation systems, phenomenology, and other content-specific aspects.
- KSM – Knowledge of the Structure of Mathematics refers to the inherent structures of the mathematical topic, such as its connections and simplifications.
- KPM – Knowledge of Mathematical Practice refers to how knowledge is developed in the process of constructing mathematical understanding.
- KMT – Knowledge of Mathematics Teaching refers to the strategies and means used to teach, including learning theories, resource limitations, and understanding of the development of students' thinking for instructional purposes.
- KFLM – Knowledge of Features of Learning Mathematics refers to learning theories and the understanding of how students learn.
- KMLS – Knowledge of Mathematics Learning Standards refers to curricular knowledge and the understanding of the sequences of content to be taught.

We also point out that this article focuses its analysis on the domain of *Mathematical Knowledge*, in which we present its categories based on the studies of Carrillo et al.[34] and Muñoz-Catalán et al[35].

[34] See footnote 30
[35] Muñoz-Catalán, M.C., Ramírez-García, M., Joglar-Prieto, N., & Carrillo-Yáñez, J. *Early childhood teachers' specialized knowledge to promote algebraic thinking through an additive decomposition task. Journal for the Study of Education and Development*, 45(1), 37–80, 2021. https://doi.org/10.1080/02103702.2021.1946640

**Figure 2** – MK Categories




Source: Adapted by the author from Carrillo et al[36].

Furthermore, regarding the specific questions raised by Carrillo et al.[37], we have the following inquiries: How to do something? When to do something? Why is something done in this way? Also included are the characteristics of the result.

Considering the theoretical framework presented, the next section will explain how the study was conducted, providing more details about the methodology.

## Research Pathways – Methodology

This study focuses on the Specialized Knowledge of Mathematics Teachers, , in which the teachers of Early Childhood Education are inserted because these also teach mathematics for children from 0 to 5 years, which can be identified through the resolution of professional formative tasks. For the development of our study, we adopted an interpretive paradigm (Bassey[38]) and a consistent investigative design known as Teaching Experiments. From the perspective of Steffe

[36] See footnote 28
[37] See footnote 30
[38] Bassey, M. (1999) *Case Study Research in Educational Settings*. Buckingham: Open University Press.

and Thompson[39] (2000), this methodology emphasizes the importance of allowing researchers to experience and students to develop mathematical reasoning through learning processes. The authors also state that without the teaching experience, it would not be possible to truly understand the various types of mathematical knowledge. Thus, this methodology proposes two areas of interest: the first is about reflecting on what mathematics should be taught to students, and the second considers how to think about the conceptual boundaries of knowledge when designing a task.

This methodology unfolds through teaching episodes. In our case, we developed three episodes: The preparation of the professional formative task among the teacher educators; The preparation of the class, addressing the concept of classification and its components; The classification task itself, aimed at developing algebraic thinking.

We are particularly interested in to identify elements of the specialized knowledge mobilized by future teachers of Early Childhood Education in initial training on algebraic thinking when they develop a professional formative classification task.

The activities included in the study involve different representations, records that promote interpretation and result analysis, participants' written reflections, their planning and organization during the task, the resources used, and the forms of intervention and mediation between the researcher and the participants.

One essential feature of Teaching Experiments is documentation, which involves written records of the entire investigative process. In this sense, the participating teachers' actions will be observed, recorded, and analyzed throughout the training process and the development of mathematical concepts related to algebraic thinking through a classification task designed for teachers in initial training for early childhood education.

The following section will address the professional formative task and how it was structured.

[39] Steffe, L. P., & Thompson, P. W. (2000) *Teaching Experiment Methodology*: Underlying Principles and Essential Elements. In: Lesh, R., & Kelly, A. E. (Eds.), *Research Design in Mathematics and Science Education* (pp. 267–307). Hillsdale, NJ: Lawrence Erlbaum Associates.

### The Professional Formative Task

We consider it an important research tool, as it brings real classroom situations closer and encourages students to reflect on different approaches and alternatives for lesson planning (Da Ponte et al.[40], Muñoz Catalán et al.[41]; Castarnado, Contreras, Muñoz Catalán, Liñán García[42]).

Joglar-Prieto, Liñán-García, and Contreras[43] reaffirm the perspectives of previous authors and define professional formative tasks as:

> When a mathematics teacher designs activities to help students understand a particular concept or procedure; when they seek ways to respond to a student's question; when they try to interpret the reasons behind difficulties with a given content; when they decide which task to implement from the textbook proposals; or when they reflect on which resource might be most effective in the classroom.

The authors highlight these examples of professional formative tasks, which help clarify how we approach the construction of our investigative task. We used video transcripts from real classroom settings to collect data on the potential knowledge held by teachers in initial training for early childhood education.

To implement this methodology, we designed a professional formative task that allowed for reflection on the specialized knowledge of early childhood education teachers regarding algebraic thinking through classification. The construction of the task took into account the specific context of early childhood teacher education for the development of algebraic thinking. Authors Alencar, Muñoz Catalán, and Liñán García (2023) conducted a study—presented in Table 1—that contributes to bridging algebraic thinking through classification with the MTSK theoretical model. We will now delve deeper into the presentation of the subdomains of mathematical knowledge.

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[40] Ponte, J. P., et al. Teachers' Professional Learning through Lesson Study. Perspectives on Mathematics Education, Campo Grande, 5 (special issue), 7–24, 2012.

[41] Muñoz-Catalán, M. C., Ramírez-García, M., Joglar-Prieto, N., & Carrillo, J. Mathematics Teacher's Specialized Knowledge to Promote Algebraic Reasoning in Early Childhood Education Through an Additive Decomposition Task. *Infancia y Aprendizaje: Journal for the Study of Education and Development*, 45, 37–80, 2022. https://doi.org/10.1080/02103702.2021.1946640

[42] See footnote 5

[43]Joglar-Prieto, N., Liñán-García, M. M., & Contreras, L. C. MTSK in Initial Teacher Education for Primary Education. In: Carrillo, J., Montes, M. Á., & Climent, N. (Eds.), *Research on Mathematics Teacher's Specialized Knowledge (MTSK): 10 Years of Development* (pp. 207–222). Dykinson, 2022.

Table 1 – Specialized Knowledge for Early Childhood Teachers on Classification

| Subdomain | Knowledge |
|---|---|
| KoT | - Knowing that in order to establish a relation, one must define the set on which it operates.<br>- The precise designation of a set by extension and by intension is essential to support the understanding of mathematical concepts.<br>- Knowing that mathematical classification, understood as the result of applying an equivalence relation, has its own characteristics: it generates equivalence classes, the elements of each class are equivalent, and it implies identification processes, meaning it is not given but depends on the equivalence relation defined over a specific set.<br>- Understanding that classification results from applying an equivalence relation to a set.<br>- Knowing the properties that a binary relation must satisfy to be considered an equivalence relation (reflexive, symmetric, and transitive properties).<br>- Knowing the characteristics of the properties of binary relations.<br>- Knowing what a binary relation is.<br>- Knowing the necessary and sufficient conditions for a binary relation to be valid and to possess its properties.<br>- Knowing that classification underlies number sense (the decimal number system).<br>- Knowing that classification enables understanding of the world by organizing it into the typology of its elements, which evolves into the understanding of this science (and any other).<br>- Knowing that different classifications exist based on: the number of criteria considered, the simultaneous application of criteria (cross-classifications), and the relationships between classes (categorization). |
| KSM | Pre-numerical knowledge as a precursor to algebraic thinking |
| KPM | - Designation as a practice of identifying the attributes of objects, serving as a precursor to classification.<br>- The role of designation requires a set to be expressed both by extension and by intension to support the comprehension of mathematical concepts.<br>- The role of symbolization and formal language in expressing a binary relation and understanding the meaning of “necessary and sufficient” in that relation.<br>- Comparing elements of the set to establish similarities and differences. |

Source: Alencar, Munhoz Catalán, and Linan Garcia[44]

[44] Alencar, E. A., Muñoz-Catalán, M. C., & Liñán-García, M. M. (2023) Approaching the Specialized Knowledge of Early Childhood Teachers for Teaching Algebraic Thinking – Classification. *Proceedings of the VI CIMTSK*. Chile, 2023.

The result of the analysis of these stages should lead to the development of a reflection on the concepts involved in the development of algebraic thinking through a classification task so that this study may later be used to reformulate initial teacher education programs and promote a better understanding of the ideas related to this topic.

Thus, in this research, we identified the specialized knowledge mobilized by students in the context of a specific formative task on classification, and we explored how future Early Childhood Education teachers are being trained at the University of Seville.

This study was conducted during the years 2023 and mid-2024. The research took place over two class sessions within a teacher training course for Early Childhood Education, with an average of 40 students. For the research, we analyzed data from three teams of five participants each in the subject "Development of Logical-Mathematical Thinking" at the University of Seville, Spain.

The students are mostly Spanish, primarily from the Seville region. They are young, with an average age of 21, studying to become Early Childhood teachers and enrolled in courses related to Mathematics Education. Therefore, these students had already engaged in prior classroom studies on classification, set construction, and the relationships between sets.

In this context, the relevant classes were observed, readings were carried out, and a formative task was developed related to algebraic thinking through classification.

As a result, we designed the task to be implemented in the selected classes, as shown in Table 2.

Table 2 – The task

| |
|---|
| Every Friday, Teacher Victoria takes her Early Childhood Education students (ages 4–5) to the "toy library" [brinquedoteca]. It's a moment the children really enjoy, as they get to play and have fun—but with the teacher's guidance and interventions, they also learn during this time.<br>The "toy library" has shelves that are accessible to the children, allowing them to independently choose the toys and also organize them before leaving the room. |

| |
|---|
| The “playroom” contains: 1 small pink teddy bear, 1 large brown teddy bear, 1 medium cream-colored teddy bear, 1 small orange teddy bear, 2 blue toy cars (one light blue and one dark blue), 1 green remote-controlled car, 1 red car, 1 pink car, 1 bus, 1 tractor, 2 medium-sized baby dolls, 3 small dolls (one with long black hair, one with short red hair, and one with medium-length blonde hair), 2 building block sets (one with rounded pieces and one with rectangular pieces), 3 tic-tac-toe games, 5 balls (2 large blue ones and 3 small orange ones), 7 costumes (2 princess/prince – pink/blue, 3 superhero/superheroine, and 2 animal costumes), 1 dollhouse with small furniture and everyday household items. |

Source: Own research

Given the task's context, we designed two situations; in this article, we will analyze one of them, including some of the guided questions, considering the limited space available in this journal. We emphasize that the questions presented in this article were designed to prompt reflection on the domain of Mathematical Knowledge – MK. The situation is presented in Table 3.

Table 3 – The task situation

| |
|---|
| Teacher Victoria: Hello, students. Today we have our activity in the “toy library”, and each of you will be able to choose three toys to play with.<br>The teacher observes that Maria chooses the pink car, the pink teddy bear, and the pink princess costume. The teacher asks: Why did you choose these toys?<br>Maria responds: Because I like pink toys.<br>At the same time, the teacher observes that Manuel chooses the light blue car, the white remote-controlled car, and the tractor. To the same question, the student answers:<br>- Because these toys move, and I can drive them.<br>- Reflection questions to justify your answers based on the situation above:<br>1. What kind of relationship is the student establishing? Express it mathematically.<br>2. Are there any toys that are not related? Which ones?<br>3. What type of relationship is this?<br>4. What other relationships could be established with this set of toys? What type would they be? |

Source: Own research

We highlight that we provided contextualization of what we consider the “playroom” within the task, as this is a school space commonly used for early childhood education activities in Brazil. Since this task originates from a postdoctoral project in which the researcher views it through a Brazilian lens, it was deemed appropriate to provide such contextualization—especially considering the differences with the Spanish educational context in which the study participants are inserted.

The task context and each situation were read in their entirety, and each question was read individually. This allowed the students time for reflection, formulation of responses, and discussion of their observations.

To analyze the data, we created a table with the responses from each team. Given the large volume of data, we selected those that stood out or contributed more deeply to the reflection on classification.

In the next section, we present the analysis conducted in this study, highlighting evidence of the aspects needed to develop initial teacher training aimed at fostering the learning of specialized knowledge for teaching algebraic thinking through classification.

**Revealed Knowledge – Analysis**

When analyzing the first reflection question, “What kind of relationship is the student establishing? Express it mathematically,” the students began searching through their previous study materials to find connections between the knowledge they had already encountered and this new situation. This was a positive observation on the part of the students, as it demonstrated their interest in figuring out how to solve the task.

When the discussion was opened, the response was: Equivalence relation. If A and B are two elements of a set, B and A are related if B has the same color as A, and A and B belong to the same set if they are the same color—pink, in this case. Through this response, it was possible to initiate our reflection on the properties of a set.

The students were able to elaborate and reflect on how these properties are necessary for the formation of a set. The written records indicated that a future teacher must understand what a set is, and the proposed situation encourages them to reflect on the set created by the student and the relationships that were established.

Thus, we selected the most interesting excerpts for discussion in this article. The selection criteria included: those that identified and described the relationships as equivalent, another group that described them as a binary relation, and another that did not identify the relationships with sufficient depth.

Table 4 – Team responses 1

| |
|---|
| Team 1 – The relationship that Maria establishes among the toys she chose is based on the color pink — all the toys are that color. In contrast, Manuel's relationship with the toys he selected is related to transportation, as they are toys he can move across any surface. Set: classroom toys (M) Relation: toys of the same color (a and b) Formula: Given two elements from set M, element a is related to b if a has the same color as b. It is an equivalence relation because it allows us to establish a connection between elements of the set that share certain characteristics — in this case, color. Furthermore, it is: Reflexive, as every element in the set is related to itself; Symmetric, because one element is related to another and vice versa; Transitive, as it allows two objects to be indirectly related through a third. All these toys are related by a common characteristic — the pink car.<br>The relationship that student Maria establishes by choosing pink-colored toys can be mathematically formulated as a preference for pink toys. In mathematical terms, it could be expressed as: Maria's Preference = {pink car, small pink bear, princess costume}. Maria chooses these toys based on her liking for the color pink, which clearly shows a preference in her toy selection in the "playroom".<br>Team 7 Set: All the toys in the playroom (M) Relation: Pink-colored toys (A and B)<br>Given a set formed by the toys in the playroom, we define the relation of pink-colored toys.<br>1. Mathematically formulated: Given two elements from set "M", element "A" is related to element "B" if "A" has the same color (pink) as "B".<br>This is a Binary Relation, since the relationships are established between two elements of the set at a time — that is, a one-to-one (binary) relationship.<br>Team 32: The relationship that Maria establishes is a preference for the color pink in the toys.<br>Mathematically, it can be formulated as:<br>Maria chooses x if x is pink. |

Source: Own research

We observe that the students are able to identify what a set is and that, through sets, it is possible to establish relationships and form subsets. Thus, we note that this question prompts future teachers to think about mathematical knowledge for teaching classifications and how these are closely linked to the development of algebraic thinking.

From this, it is inferred that teachers must know what a set is and the possible relations that can be established within it in order to better develop their pedagogical practices. Specific characteristics were observed, and it was found that when students are asked to mathematically describe set relations, some of them (Team 1) are able to recognize that an equivalence relation must satisfy the reflexive, symmetric, and transitive properties. Other students (Team 7) identify only the binary relation without going deeper into its characteristics, which reveals a superficial understanding of sets. There are also those (Team 32) who struggle to reflect on the topic and are still in the early stages of learning; they are able to formulate the relation mathematically, but without deeper conceptual understanding.

This question further leads us to reflect: why does a teacher need to know the properties of equivalence in order to teach preschool children? The answer is that knowing what a set is, how it is related, and what its properties are gives teachers a stronger foundation for teaching and for designing learning situations for students—such as the example we presented involving toy classification.

To achieve this, a game-based scenario must be closely linked to the learning process, and for the teacher to implement it effectively, they must understand how it can be used and what knowledge it aims to develop. Thus, understanding sets as a whole—and how their properties are identified within mathematical knowledge—relates to Knowledge of Teaching (KoT), as described by Carrillo (2018 et al.), specifically within the category of Definitions, Properties, and Foundations. Furthermore, prompting students to think about the idea and characteristics of what constitutes a set is associated with pre-numerical knowledge, which is crucial for the development of algebraic thinking. This knowledge is also part of the Structure of Mathematics Knowledge (KSM), specifically in the category of Transversal Connections - those connections not tied to a single mathematical content area, but which support and inform learning in specific domains.

We also observed Knowledge of Mathematical Practices (KPM), particularly in the category Role of Symbols and the Use of Formal Language, when we identified the role of symbolization and formal language in expressing the student's criteria mathematically.

When we asked the students to reflect on the question: Are there toys that are not related? Which ones? We noted that there was no reflection on the differences between types of classifications and that there are various ways to classify—as we can see in the responses shown in the table.

To reflect on this article regarding the knowledge of pre-service teachers, we selected two responses: one that does not identify the relationship between the toys and another that sees the relationships as dependent on the way the classification is made.

Table 5 – Team responses 2

| |
|---|
| Team 6 – Yes, there are toys that are not related to each other, because the relationship is established when they have the same color. Therefore, a pink toy is not related to a toy that is not pink. |
| Team 17 – It depends on the relationship you establish. If the relationship you make is that A and B are pink and the others are not, then the relation is based on the criterion of being pink or not pink. |

Source: Own research

We observed that some teams describe the set relationships as not possible, making inferences about the criteria used, while others simply state that it depends on what is chosen as part of the set. In this way, we can see how future early childhood teachers understand that within a set, it is possible to define another subset depending on the criteria applied. Reflecting on this helps us understand the extensions of a set and how one can compare and identify their properties. We notice the presence of Knowledge of Topics (KoT) in the category Definitions, Properties, and Foundations, as students begin to understand that there are different criteria for establishing relationships, which can occur within a larger set—as in the case of the toys.

In this question, we also identify Knowledge of Mathematical Practices (KPM), specifically in the Problem Processes category, as we encourage students to think about different ways to compare, establishing similarities and differences.

Questions 3 and 4 lead us to think: What type of relation is this? What other relations could be established with this set of toys? What type would they be?

These questions are related to the previous answers, as we noticed that some teams once again refer to properties of equivalence. Therefore, we selected one example from each team with different responses for analysis in this article.

Table 6 – Team responses 3

| |
|---|
| Team 9 – The given relation is binary because it compares one toy to another, and if a third toy is added, it only needs to be compared with one of them to determine whether or not it is related.<br>Within this binary relation, we find an equivalence relation, since it satisfies the reflexive, symmetric, and transitive properties ("being equal to"): – Reflexive, because any element in the set is related to itself, since it has the same color as it.<br>– Symmetric, because if "a" is related to "b" for having the same color, then "b" is related to "a" for having the same color. – Transitive, because if "a" is related to "b" for having the same color, and "b" is related to "c" for having the same color, then "a" is related to "c" for also having the same color. |

The students in early childhood teacher training bring reflections showing that the relation is binary and an equivalence relation because it meets the required properties. Team 9 does not go deeper by mentioning other possible relationships with the toy set, which shows that they are very focused on the properties and the studies previously done in class.

It is interesting how Team 6 brings a conclusion involving anti-reflexivity.

Table 7 – Team responses 6

| |
|---|
| Team 6 – "Having the same pink color"<br>This type of relation does not exist in which the properties are:<br>Antireflexive: Because NOT all elements in the set are related to themselves, since not all toys in the classroom are pink—there are toys of other colors.<br>Symmetric: Yes, whenever one element is related to another, the second is certainly related to the first.<br>Transitive: Yes, because there are relations involving 3 elements → "A" is related to "B", and "B" is related to "C", so "A" is related to "C".<br>Having the pink color does not establish an order relation; it has no name (it's a special case). |

Source: Own research

There are also teams that do not deal with mathematical relations; they do so in a superficial way, as we can see with Team 2.

Table 8 – Responses of team 2

| |
|---|
| Team 2 – The relation established in the situation described in the "toy library" between the toys selected by the students Maria and Manuel is subjective and personal. Maria chooses the pink toys because she likes them, while Manuel chooses toys that move and that he can drive. These choices are based on the individual preferences of each student, reflecting a subjective relationship between the toys and the children's personal preferences. |

Source: Own research

As for the responses to Question 4, we generally see answers like the one given by Team 2.

Table 9 – Responses of team 2

| |
|---|
| Team 2 – Relation by type of toy: The toys could be grouped according to their type, such as balls, dolls, vehicles, building games, costumes, among others. This type of relation would be a categorization by function.<br>- Relation by color: The toys could be grouped according to their color, such as pink toys, blue toys, orange toys, etc. This relation would be a classification by visual attribute.<br>- Relation by size: Toys can be organized according to their size, such as small, medium, and large toys. This relation would be a classification by dimension.<br>- Relation by theme: The toys could be grouped according to their theme, such as princess toys, superhero toys, vehicles, and so on. This relation would be a classification by theme or category. |

Source: Own research

Based on the above, we identified that the teachers in initial training in this study reflect on the tasks presented and are able to develop reflections that contribute to their specialized knowledge.

We consider that the evidence related to Mathematical Knowledge (MK), its subdomains, and how reflection on this aspect can encourage further research in the area is significant. The development of this task highlighted the importance of prior preparation of students in initial teacher education programs for Early Childhood Education, including studies on the relationships involved in classification and what characterizes a set. Therefore, we see throughout the analysis section the revealed knowledge is focused on mathematical knowledge - KoT, with specificity in the category Definitions, properties and foundations. In addition, in the knowledge of the Structure of Mathematics - KSM, category of transverse connections. In the Knowledge of Mathematical Practices - KPM, in the category Role of Symbols and the use of formal language and in the category Problem Processes.

Moreover, designing tasks based on everyday contexts that connect to pedagogical practice allows for a closer approach to the knowledge embedded in real situations.

## Final Considerations

As we developed this research using a task designed for early childhood education students to reflect on classification relationships—and how these allow future teachers to think about algebraic thinking—we were encouraged to draw some conclusions about the development process of algebraic thinking in early childhood.

The main aspect necessary for developing initial teacher education that fosters the learning of specialized knowledge to teach algebraic thinking through a classification task is prior study of what a set is and what relationships are involved in making a classification, contained mathematical knowledge - KoT. We believe that, after carrying out the tasks proposed in this research, students are encouraged to reflect on children's everyday experiences, which may give greater meaning to their learning. We also note that other knowledge are important as the Knowledge of the Structure of Mathematics - KSM and the Knowledge of Mathematical Practices - KPM.

Furthermore, we consider that bringing tasks that encourage future early childhood teachers to reflect on mathematical knowledge in a classification situation—as a way to develop algebraic thinking—is something that can lay the groundwork for further research in the field, as well as for the reformulation of teacher education programs.

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